LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FOURTH SEMESTER – **APRIL 2013**

MT 4502/MT 4500 - MODERN ALGEBRA

Date: 02/05/2013 Dept. No. Max. : 100 Marks
Time: 1:00 - 4:00

PART – A

ANSWER ALL QUESTIONS: (10 x 2 = 20)

1. Define finite and infinite set.
2. Define partially ordered and totally ordered set.
3. Define order of an element of a group.
4. State Lagrange's theorem.
5. Define automorphism of a group with an example.
6. Define odd and even permutations.
7. Define a ring with an example.
8. Define an integral domain.
9. What is a Gaussian integer?
10. State Unique factorization theorem.

PART – B

ANSWER ANY FIVE QUESTIONS: EACH QUESTION CARRIES EIGHT MARKS: (5 x 8 = 40)

11. State and prove the cancellation laws in a group.
12. Prove that every subgroup of a cyclic group is cyclic.
13. Show that the intersection of two normal subgroups is again a normal subgroup.
14. State and prove fundamental homomorphism theorem.
15. State and prove second isomorphism theorem.
16. Prove that every finite integral domain is a field.
17. Prove that every field is a PID.
18. Let R be a commutative ring with unity and M an ideal of R. Then prove that M is a maximal ideal of R if and only if R/M is a field.

PART – C

ANSWER ANY TWO QUESTIONS: (2 x 20 = 40)

19. a. Show that every group of order four is abelian.
    b. If H and K are finite subgroups of a group G, then prove that $o(HK)=\dfrac{o(H)o(K)}{o(H\cap K)}$.
20. a. State and prove Lagrange's theorem.
    b. State and prove Cayley theorem.
21. a. Let R be a commutative ring with unit element whose only ideals are (0) and R itself. Then prove that R is a field.
    b. If p is prime then prove that Z_p is a field.
22. a. Prove that the characteristic of an integral domain D is either zero or a prime number.
    b. Let R be a commutative ring with unity and P an ideal of R. Prove that P is a prime ideal of R if and only if R/P is an integral domain.

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